## The Mind vs. Logic: Aristotle and Zadeh

Leonid I. Perlovsky

Air Force Research Lab., 80 Scott Rd., Hanscom AFB, MA 01731 Tel. 781-377-1728; e-mail: Leonid.Perlovsky@hanscom.af.mil

## ABSTRACT

Fuzzy logic invented by Zadeh is usually counterposed to the usual logic invented by Aristotle. This paper argues that Aristotle did not consider logic as a fundamental mechanism of the mind, rather he considered the working of the mind to be more similar to fuzzy logic.

**KEYWORDS:** Aristotle, Zadeh, logic, fuzzy logic, mind.

Aristotle, I heard you are writing books now. Are you going to make our secret knowledge public? from a letter by Alexander

Alexander, do not worry: nobody will understand. from a reply letter by Aristotle

In a seminal paper in 1965 Lotfi Zadeh introduced fuzzy logic [<sup>1</sup>] to describe, mathematically, the imprecision characterizing the mind's reasoning about the real world. Fuzzy logic emphasizes that every statement is a matter of degree. This is widely believed to be a sharp break with traditions of classical Aristotelian logic. It is interesting therefore to note that the original Aristotelian thinking might have been closer to the fuzzy logic of Zadeh than is usually appreciated. Aristotle closely tied logic to language. He emphasized that logical statements should not be formulated too specifically, otherwise meaning might be lost. He argued, "language contains necessary means for appropriate formulation of logical statements" and "common sense must be used to do it" [<sup>2</sup>]. However, Aristotle also formulated the "law of excluded middle", which contradicted the uncertainty of language. For more than two thousand years, the legacy of Aristotle has contained this unresolved contradiction.

The contradiction was noted in the 19<sup>th</sup> century by George Boole, who thought that logic could be improved by excluding any uncertainty which is a part of causal language. A great school of logic formalization emerged, promising in the eyes of many to completely and forever formalize scientific discourse. Prominent mathematicians contributed to the development of formal logic, including George Boole, Gottlob Frege, Georg Cantor, Bertrand Russell, David

Hilbert, and Kurt Gödel. Logicians cast aside the uncertainty of language and founded formal mathematical logic based upon the law of excluded middle. Most physicists today agree that the exactness of mathematics is an inseparable part of physics, but formal logicians went beyond this. Hilbert developed an approach named formalism, which rejected the intuition as a part of scientific investigation and thought to define scientific objects formally in terms of axioms or rules. The physical reality of the world, he thought, could be equally represented by any set of axioms that did not contradict physical data.

Hilbert was sure that his logical theory also described mechanisms of the mind: "The fundamental idea of my proof theory is none other than to describe the activity of our understanding, to make a protocol of the rules according to which our thinking actually proceeds" [<sup>3</sup>]. In the 1900s he formulated his famous Entscheidungsproblem: to define a set of logical rules sufficient to prove all past and future mathematical theorems [<sup>4</sup>]. This entailed the formalization of scientific creativity and the entire human thinking.

Almost as soon as Hilbert formulated his formalization program, the first hole appeared. In 1902 Russell exposed an inconsistency of formal procedures by introducing a set R as follows: R is a set of all sets, which are not members of themselves [ $^5$ ]. Is R a member of R? If it is not, then it should belong to R according to the definition, but if R is a member of R, this contradicts the definition. Thus, either way we get a contradiction. This became known as the Russell's paradox. Its joking formulation is as follows: A barber shaves everybody who does not shave himself. Does the barber shave himself? Either answer to this question (yes or no) leads to a contradiction. This barber, like Russell's set, can be logically defined but cannot exist. For the next 25 years mathematicians where trying to develop a self-consistent mathematical logic, free from the paradoxes of this type. But in 1931 Gödel proved that it is not possible [ $^6$ ]; formal logic was inconsistent, and self-contradictory.

Today we know that logic is not a fundamental mechanism of the mind [<sup>7</sup>]. Logical or approximately logical reasoning is a result of adaptive dynamic processes. These processes in the mind start from fuzzy vague states-representations and evolve toward approximately logical, low-fuzzy states-representations adapted to the concrete reality around us. We can get an idea about fuzzy states-representations if we close our eyes and imagine a well-known object, like a chair. The imagined chair is much less crisp and more fuzzy than an "actual" chair perceived with opened eyes. The "actual" chair is a result of dynamic adaptation, or a state of resonance between top-down imagination signals and bottom-up sensory signals. This process is amazingly close to Emmanuel Kant's description of understanding as a result of interplay between sensing and imagination [<sup>8</sup>].

The founders of artificial intelligence, including Allan Newell and Marvin Minsky, thought that formal logic was sufficient [<sup>9</sup>] and that no specific mathematical techniques would be needed to describe the mind [<sup>10</sup>]. Why did many fine mathematicians believe that the mind is logical, despite the obvious observations that this is not so, and why did many continue believing in logic even after Gödel? What is the explanation for such illogical insistence on logic?

Belief in logic has deep psychological roots related to the functioning of the mind. A major part of any perception and cognition process is not accessible to consciousness directly. Individual neuronal firings are not accessible to consciousness. We are not conscious about early stages of dynamic adaptive processes of perception and cognition, when the mind states-representations are vague and fuzzy. Our consciousness can only access later stages, when states-representations become low-fuzzy, and approximately logical. As states-representations of the mind become more crisp, they also become more directly accessible to consciousness. For this reason many people, including prominent mathematicians, believed in logic.

Returning to Aristotle, we note that he saw logic as an instrument of public discourse, and a way to correctly argue for conclusions which have been already obtained by other means. This is clearly seen, for example, from "Rhetoric for Alexander" [<sup>11</sup>], where he lists logical arguments that should be used in public speeches as needed for, or against, dozens of political issues. For example, such issues might include declaring war or making piece, signing treaty or refusing it, trusting or mistrusting a witness, whether or not to use torture to obtain trustworthy evidence, etc. Aristotle provided exact logical ways to argue both for, or against, any issue. Never had he given the impression that logic was a mechanism of obtaining truth. Logic, to him, was a tool of politics and not of science. I would extend Aristotelian arguments for scientists: use logic when writing a paper, but not when solving a new problem. When Aristotle was seeking an explanation of human thinking, he developed a theory of Forms  $[^2]$ . The main tenets of this theory are that perception and cognition are processes in which "a priori Forms meet matter." This process is the foundation for all our experience, and it creates concepts with which our mind thinks and perceives individual objects and situations. A priori Forms exist in our minds in a different state than the 'final' state of concepts used for thinking and perception. Aristotle called the initial states of a priori Forms "potential states" or "potentialities." Their final states (of concepts of perception and cognition), after they "met matter," he called "actual states" or "actualities." He emphasized that whereas actualities obey the rule of excluded middle, potentialities do not [12]. It is my opinion that if Aristotle knew fuzzy logic, he would say that the a priori Forms are fuzzy, whereas the 'final' concepts consciously used by the mind are crisp (or low-fuzzy), and the process by which "a priori Forms meet matter" is given by fuzzy dynamic logic [7].

Let us repeat the simple experiment described above: close your eyes and imagine a familiar object. The imagination is not as crisp and clear as your perception of the object with opened eyes. Imagined objects are created in our visual cortex when stimulated by signals from the internal mind's representations. This simple experiment reveals an intriguing property of the mind. We can actually observe the fuzziness of the mind's representations. These representations serve as the basis for perception of simple objects with opened eyes. There is a drastic difference in the degree of fuzziness of imagined and perceived objects. Let us ask ourselves, what is a degree of fuzziness of states-representations corresponding to abstract concepts, such as "rationality," "love," "meaning of life"? Our eyes cannot directly perceive these abstract concepts, because they are not objects that can be observed. Should we assume that abstract concepts have to be more vague and fuzzy than imagination? This seems to contradict our direct experience of abstract concepts. When we talk about rationality, or family, or trust, or many other abstract notions, we think we understand what we are talking about, and our mind is not totally vague and fuzzy. This contradiction is due to the difference between the statesrepresentations of cognition and language. By five to seven years of age a child can speak about many abstract concepts, including virtually the entire content of culture. But this does not mean that a seven year old "really knows" what he or she is talking about. A mathematical description of this mechanism is given in [<sup>13</sup>]: the mind concept-representations consist of two parts, cognitive and linguistic. We receive the linguistic part of our concepts "ready-made" from the surrounding language. By seven years of age linguistic representations are low-fuzzy, crisp, and conscious. This gives the appearance to a seven-year old consciousness, and often to surrounding people, that he or she knows what they are talking about. But it takes the rest of ones life to make cognitive representations as concrete and conscious as linguistic are. Many four-year-olds can speak fairly well about good guys and bad guys, but who at the age of forty or even eighty can

claim that his cognitive representations of good and bad are equally crisp? The nature of good and evil remains an unresolved philosophical issue after millennia of debates. Describing this situation in terms of logic, we would say that many cognitive states-representations of the mind are fuzzy, although we talk about them using approximately logical, low-fuzzy, linguistic descriptions. And this conundrum involving fuzzy and crisp, as we all know, contains infinite possibilities for scientific discourse. Evolution of cultures and science bridges the gap between unconscious-fuzzy and conscious-crisp, however infinite possibilities still remain for scientific development.

## REFERENCES

- <sup>6</sup> Gödel, K. (1986). Kurt Gödel collected works, I. (Ed. S.Feferman at al). Oxford University Press.
- <sup>7</sup> Perlovsky, L.I. (2006). Fuzzy Dynamic Logic. New Math. And Natural Comp., **2**(1), pp.43-55.
- <sup>8</sup> Kant, I. (1790). The Critique of Judgment, tr. J.H.Bernard, Prometheus Books, Amherst, NY.
- <sup>9</sup> Newell, A. (1983). Intellectual issues in the history of artificial intelligence. In The study of information, ed. F.Machlup & U.Mansfield, J.Wiley, New York, NY.
- <sup>10</sup> Minsky, M. (1988). The Society of Mind. MIT Press, Cambridge, MA.
- <sup>11</sup> Aristotle, (IV BCE). Rhetoric to Alexander. In the Complete Works of Aristotle, Ed.J.Barnes, Bollingen Series, 1995, Princeton, NJ.
- <sup>12</sup> Aristotle, (IV BCE). Topics. In the Complete Works of Aristotle, Ed.J.Barnes, Bollingen Series, 1995, Princeton, NJ.
- <sup>13</sup> Perlovsky, L.I. (2006). Toward Physics of the Mind: Concepts, Emotions, Consciousness, and Symbols. Phys. Life Rev. 3(1), pp.22-55.

<sup>&</sup>lt;sup>1</sup>Zadeh, L.A. (1965). Fuzzy sets, Information and Control, 8, pp. 338-353.

<sup>&</sup>lt;sup>2</sup> Aristotle, (IV BCE). Metaphysics, In the Complete Works of Aristotle, Ed.J.Barnes, Bollingen Series, 1995, Princeton, NJ.

<sup>&</sup>lt;sup>3</sup> Hilbert, David. (1928/1967). The foundations of mathematics. In J. van Heijenoort, Ed., From Frege to Gödel. Cambridge, MA: Harvard University Press, p.475.

<sup>&</sup>lt;sup>4</sup> Hilbert, D. (1900). See in The Honors Class: Hilbert's Problems and Their Solvers, Ben Yandell, AK Peters Ltd, 2003, Natick, MA.

<sup>&</sup>lt;sup>5</sup> Russel, B. & Whitehead, A.N. (1908). *Principia Mathematica*. Cambridge Univ. Press, 1962, Oxford, England.