A Network Model of Rational versus Irrational Choices on a Probability Maximization Task

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Abstract—**Humans have a drive to maximize knowledge of the world, yet decision making data also suggest a contrary drive to minimize cognitive effort using simplifying heuristics. The trade-off between maximizing knowledge and minimizing effort is modeled by simulation of a challenging decision task. The task is to choose which of two gambles has the highest probability of success when the alternative with higher success probability also has lower success frequency.**

I. INTRODUCTION

Because of the large number of decisions we must make and the shortness of time for many of them, humans have evolved a set of neurobehavioral processes that provide for efficient decision making. The work of Reyna and Brainerd (see, e.g., [1-4]) suggests that we encode traces of items we learn, including those we choose from, in two different ways: *verbatim* and *gist* encoding. As we become adults, or as we become familiar with a domain of knowledge, our predominant encoding gradually switches from verbatim to gist.

The ability to grasp the gist of a problem, and ignore relatively minor details, facilitates not only our efficiency, but also our ability to recognize the problem as similar to some others we have encountered before and thereby draw on our memory of those other problems. Yet gist processing is also a main source of heuristics that can sometimes lead to errors. In particular, as Reyna and Brainerd note, we sometimes encode the wrong gist for a particular problem.

Complex reasoning capabilities often involve decisions between competing gists. A variety of decision making tasks tend to evoke two or more competing rules, one of which is normatively superior to the others. One example is a task that involves choosing a larger probability versus a larger frequency of either a gain or a loss (frequency in the absolute, not relative, sense; see, e.g., [4-7]).

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Yamagishi [5] found that the majority of their participants judging the riskiness of various causes of death were more influenced by the described numerosity of deaths than by the probability of death. For example, they rated cancer as riskier when it was described as killing 1,286 out of 10,000 people than when it was described as killing 24.14 out of 100 people.

The phenomenon whereby the same probability is experienced as larger if it comes as a ratio of two larger numbers has been called *ratio bias* (e.g., [7]). Pacini and Epstein [7] found that many of their participants seemed to be aware of their ratio biases, but were conflicted between emotional and rational influences on their choices.

II. THE RATIO BIAS PARADIGM

To limit our theoretical domain, we simulated a particular version of the frequency/probability decision task due to Denes-Raj and Epstein [6]. The aim of our work is to generalize from modeling this restricted case to a more general model of rule selection; in particular, selection between a simple, readily available, but nonoptimal rule and a more complex but more accurate rule.

Participants in the Denes-Raj and Epstein experiment were assigned randomly either to a *win condition* or a *loss condition*. In the win condition, they were shown two bowls containing red and white jellybeans, told they would win a certain amount of money if they randomly selected a red jellybean, and instructed to choose which bowl gave them the best chance of winning money. In one of the bowls, there were always a total of 10 jellybeans out of which 1 was red. In the other bowl, there were a total of 100 jellybeans out of which some number greater than 1 but less than 10 were red. Hence, choice of the bowl with a larger frequency of red jellybeans was always nonoptimal, because the probability of drawing red from that bowl was less than 1/10. The loss condition used the same two bowls, but the participants were told they would lose a certain amount of money if they selected a red jellybean, so the bowl with more jellybeans was the optimal choice.

Figs. 1 and 2 show percentages of nonoptimal responses in both win and loss conditions. "Nonoptimal response size" in that graph means the difference between the chosen option and 10 out of 100 which was equivalent to 1 out of 10; that is, 1 represents the choice of 9 out of 100 over 1 out of 10, 2 represents the choice of 8 out of 100, et cetera.

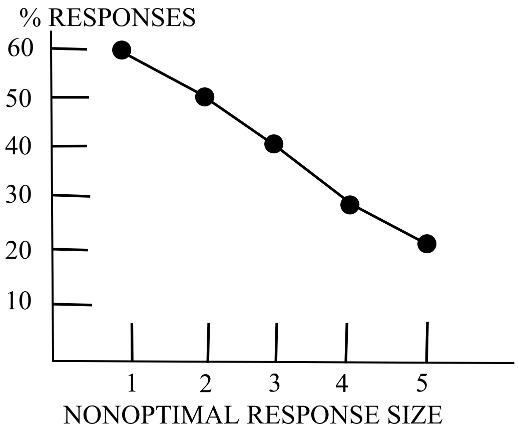

Fig. 1. Experimental results [6] on percentages of time the higher-frequency, lower-probability alternative was chosen on win trials. See the text for explanation of nonoptimal response size. (Adapted from [6] with permission of the American Psychological Association.)

In the win condition, the majority of participants made the nonoptimal choice when the choice was 9 out of 100 (nonoptimal response size 1) versus 1 out of 10, and about a quarter still chose 5 out of 100 (nonoptimal response size 5) over 1 out of 10. Larger response sizes are not shown in the graph, but the authors remarked that no participant chose 2 out 100 over 1 out of 10. In the loss condition, the pattern of drop-off was similar but there were significantly fewer nonoptimal choices. The authors explained the difference between win and loss conditions by noting that the loss condition involves negative affect, which leads to more careful (and therefore, at least sometimes, rational) consideration of alternatives.

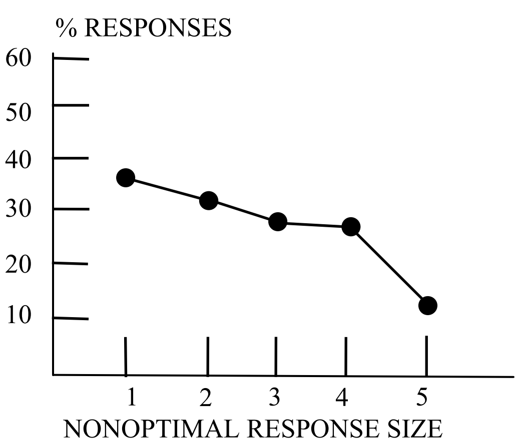

Fig. 2. Experimental data from [6] for the loss condition. (Adapted from [6] with permission of the American Psychological Association.)

III. THE NETWORK MODEL

Our model is based on assumed functions of different areas of the prefrontal executive system, notably the anterior cingulate cortex (ACC) and dorsolateral prefrontal cortex (DLFPC). One of us (DSL) is collaborating with Dr. Daniel Krawczyk (Center for Brain Health, University of Texas at Dallas) on brain imaging experiments designed to test our model by studying activations of these two regions for different classes of decision makers on the probability/frequency task.

First let us describe three basic types of decision makers (DMs) on this task (with the caveat that these characterizations may either be personality-dependent, task-dependent, or both):

(a) DMs who choose, say, 8-in-100 over 1-in-10 and are not aware of any reason to do otherwise;
(b) DMs who choose 8-in-100 over 1-in-10 but verbalize a numerical reason for making the opposite choice;
(c) DMs who correctly choose 1-in-10 over 8-in-100.

Our hypothesis is that types (b) and (c) will show more ACC activation than type (a), and type (c) will show more DLPFC activation than either type (a) or (b). The hypothesis about ACC is based on that region's role both in detection of potential errors and in response conflicts [8, 9]. Hence, decision makers who note a potential conflict between competing gists [1-4] such as frequency and probability (or, as in Epstein's studies, "rational" and "emotional" criteria) should tend to show high activity in that region. The hypothesis about DLPFC is based on a large body of research implicating that area (especially in the left hemisphere) in complex working memory manipulation. Recent fMRI studies have shown DLPFC activity correlates with accurate stimulus-response contingencies [10] and rule-based response selection [11].

Our neural network theory incorporates differences between individuals in two parameters representing ACC and DLPFC function. One or another of these parameters could correlate with the psychological construct of *need for cognition* developed by Cacioppo and Petty [12, 13]. This construct has been characterized as follows:

> People high in need for cognition possess high intrinsic motivation to engage and enjoy effortful cognitive activities, they are able to recall more relevant information about the task, to analyze accurately the quality of arguments … when compared with individuals low in the need for cognition ([14], pp. 142-143).

The decision between the two alternative gambles is based on either one of two rules, a *heuristic rule* based on frequencies and a *ratio rule* based on probabilities. The "ACC" parameter, called α, determines the likelihood of choosing the ratio rule for a given pair of gambles. If the ratio rule is chosen, the "DLPFC" parameter, called δ, determines the probability that the optimal response is made. The choice between rules is suggested by the modeling framework of *neural modeling fields (NMF)* [15]. The basis of NMF is that conceptual models of the same events in the world compete with one another and as one ascends in the network hierarchy, these models become both more complex and more accurate representations.

The heuristic rule is defined by the frequencies of the alternatives and the fuzzy concept of “much larger than 1,” which is close to one of Zadeh’s original examples of a fuzzy set [16]. It is assumed that in the absence of sufficient ACC activity, decision is controlled by the emotional center in the amygdala using a rule "choose k out of 100 over 1out of 10 if the numerator k is much larger than one." The denominator may affect the values of the fuzzy set that rule generates but is then ignored. The fuzzy membership function of k in the “much larger” category, called $\psi(k)$, is set to be a ramp function that is linear between the values 0 at $k = 3$ and 1 at $k = 13$, hence,

$$\psi(k) = \begin{cases} 0, k < 3 \\ .1(k-3), 3 \le k \le 13 \\ 1, k > 13 \end{cases} \tag{1}$$

The ACC parameter α, across trials (representing all choices made by all participants in the experiment), varies uniformly over the interval [0, 1]. If the function $\psi(k)$ of (1) is less than or equal to α, the heuristic “much larger” rule is chosen. Otherwise, a rule of “largest ratio of numerator to denominator” is chosen, that is

$$\begin{cases} heuristic\ \ chosen\ \ if\ \ \alpha \le \psi(k) \\ ratio\ \ rule\ \ chosen\ \ if\ \ \alpha > \psi(k) \end{cases} \tag{2}$$

But that ratio rule, while more likely to lead to the choice of the higher probability alternative than the heuristic rule, does not guarantee the higher probability alternative (in this case, 1 out of 10) will be chosen. This is because of the Gaussian tuning curves of numerosity detectors in the parietal cortex [17, 18], which have been suggested as a neural substrate for imprecise numerical “gists” [19].

Our network algorithm assumes that the numerators and denominators of both alternatives (k, 100, 1, and 10) each activate a Gaussian distribution of parietal numerosity detectors. Hence, before the ratios are computed and compared, each of those numbers is multiplied by a normally distributed quantity with mean 1. To obtain the standard deviation of this variable normal multiplier, we assume (based on the DLPFC’s working memory functions) that DLPFC inputs to parietal cortex sharpen the tuning of these numerosity detectors. Hence, higher dorsolateral activity should lead to a smaller standard deviation and thereby greater accuracy of relative probability estimations. Specifically, the standard deviation of each normal quantity is proportional to 1-δ, with δ being the DLPFC parameter. We found that a value of .1(1-δ) was fairly accurate in reproducing the data of Fig. 1 for the win condition. Across trials, we assumed that δ is normally distributed with mean .5 and standard deviation .25: the wide deviation relative to the mean mimics the wide range in human need for cognition [12, 13].

Hence if the ratio rule is chosen, the nonoptimal choice of k out of 100 over 1 out of 10 is made if the *perceived* ratio of red jellybeans to total jellybeans is higher in the first alternative than in the second alternative. Based on the Gaussian perturbations of numerators and denominators described above, this means that a nonoptimal choice is made if and only if

$$\begin{cases} \dfrac{k(1+\phi r_1)}{100(1+\phi r_2)} > \dfrac{(1+\phi r_3)}{10(1+\phi r_4)} \\ where\, \phi = .1(1-\delta) \\ r_i, i = 1,2,3,4\ \ are\ \ unit\ \ normals \end{cases} \tag{3}$$

Thus far we have described the simulation algorithm as a mathematical process without reference to a neural network diagram. However, ratios such as shown in Eq. (3) can be interpreted as steady states of a shunting on-center off-surround network, as follows. Present the two alternatives as inputs to the network shown in Fig. 3. Assuming perfect accuracy of numerical perceptions (otherwise the values k, 100, 10, and 1 in the circles of that Fig. are replaced by their normally perturbed values), the activity of the node u_1, representing the utility (i.e., reward value) of the bowl with k red out of 100, can be described by a nonlinear shunting differential equation with excitatory input k and inhibitory input 100-k:

$$\frac{du_1}{dt} = -\lambda u_1 + (1-u_1)k - u_1(100-k) \tag{4}$$

where λ is a decay rate. If we assume that time is short enough that the decay rate λ is 0, set the derivative in (4) to be 0 and solve for u_1, we obtain as a steady state value

$$u_1 = \frac{k}{100},$$

which is exactly the probability of drawing a red jellybean from Bowl 1. Similarly, the steady state value of u_2 comes out to be 1/10, the probability of drawing a red jellybean from Bowl 2. The mutual nonrecurrent inhibition between those nodes leads to the choice of whichever bowl has the larger u_i value.

Returning to the algorithm, by Eq. (2), since α is uniformly distributed across [0,1], the probability of the heuristic rule being chosen for a given value of k is 1-$\psi(k)$ as defined by Eq. (1). Assuming that the heuristic rule does not engage the ACC and thereby always leads to a nonoptimal choice, the probability of a nonoptimal choice becomes

$$\psi(k) + (1-\psi(k))r(k) \tag{5}$$

where $r(k)$ is the probability that the inequality (3) holds, that is, the probability of a nonoptimal choice if the ratio rule is chosen.

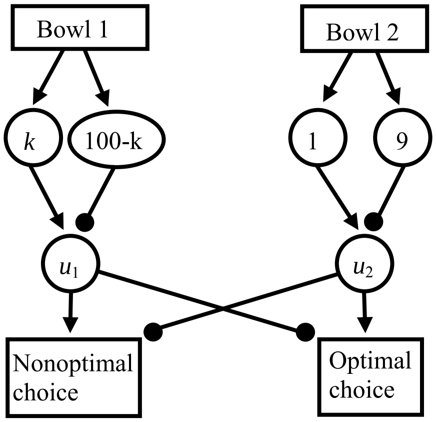

Fig. 3. Network representing choice between k-out-of-100 and 1-out-of-10 assuming use of the ratio rule. k out of 100 is interpreted as k good and 100-k bad; similarly, 1 out of 10 is 1 good and 9 bad. Probabilities of drawing red in each bowl are steady state values of Eq. (4) for node activity at u_1 and its analog at u_2, representing "utilities" of the two bowls. Arrows denote excitation, filled circles inhibition.

Hence (5) was graphed as a function of nonoptimal response size (which = 10-k) in order to simulate the data curves in Figs. 1 and 2. This was done via Monte Carlo simulations in MATLAB R2006a, the program being run 1000 times with δ allowed to vary normally about a mean of .5 with standard deviation .25.

Fig. 4 shows the results of this simulation of the win condition in the experiment of [6]. The simulation fits the data of Fig. 1 fairly closely, going from a maximum of over 60% nonoptimal responses for $k = 1$ to slightly above 20% nonoptimal responses for $k = 5$.

For the loss condition, the same program was run except that the probability of $\psi(k)$ of staying with the heuristic rule was cut in half, leading to the fraction of nonoptimal responses being

$$.5\psi(k) + (1-.5\psi(k))r(k) \quad (5a).$$

In other words, the loss condition was assumed to engage the ACC more than the win condition. The resulting graph, shown in Fig. 5, fits the data of Fig. 2 fairly closely.

IV. DISCUSSION

Can we distinguish using brain imaging those decision makers who use optimal versus nonoptimal rules in probability tasks? In a less cognitively demanding task, DeMartino et al. [20] found differences in brain activation patterns between heuristic-bound decision makers and DMs that violate typical heuristics. This study used a monetary analog of Tversky and Kahneman's "Asian disease" problem, involving a choice between a sure thing and a gamble that could be framed either as gains or losses. As in the Asian disease problem, the majority of participants chose the sure option with a gain frame and the gamble option with a loss frame. Yet significant minorities of participants chose the gamble with a gain frame or the sure option with a loss frame, in violation of the usual heuristics. fMRI measurements showed that the heuristics-violators had more activation than the heuristics-followers both in ACC and in the orbitofrontal cortex (OFC). Conversely, the heuristics-followers had more activation in the amygdala, the subcortical area most involved with primary emotional experience.

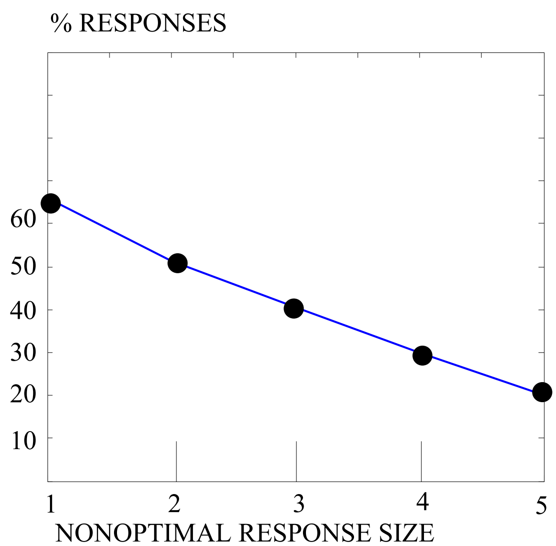

Fig. 4. Results of our simulation of the win condition of the Denes-Raj and Epstein [6] experiment.

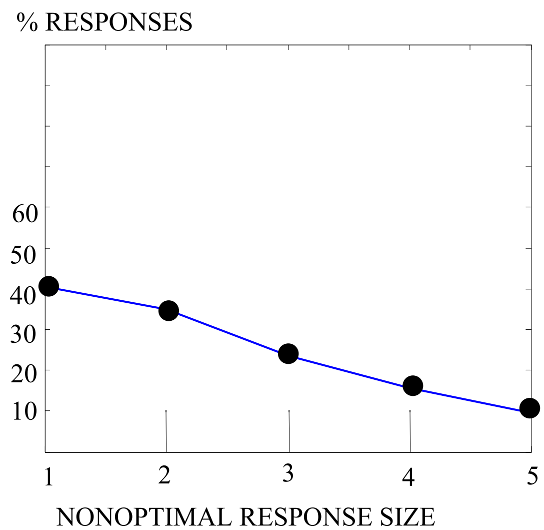

Fig. 5. Results of our simulation of the loss condition of the Denes-Raj and Epstein [6] experiment.

The probability/frequency task modeled here is more cognitively demanding than the task of DeMartino et al. [20]. Hence we expect that DLPFC, the prefrontal area involved in the most complex cognitive processing, should play a larger role in the probability/frequency task. The difference in function between these two prefrontal regions is believed to be primarily one of abstraction in their representations [21]. That theory is based in part on monkey data implicating OFC in processing shifts of simple reward contingencies and DLPFC in processing shifts of rule types [22].

The frequency/probability network discussed here is a step toward network modeling of the more general process of deciding appropriate rules for decision tasks, when the potential rules can be at any one several cognitive levels [11]. Recent fMRI studies, combined with neural network theories, suggest that ACC is sensitive to the level of complexity of tasks, or in other words, to the potential for error if the wrong rule is chosen [23]. If the task is determined to be relatively effortful, the ACC then "recruits" other brain regions required for processing task details. This often includes activation of DLPFC, and may also include signals to midbrain nuclei that produce modulatory neurotransmitters [24]. The detailed structure and function of network connections between all these executive-related brain regions is still largely unknown, and we hope the algorithmic framework presented here can ultimately lead to data-driven hypotheses about those connections.

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